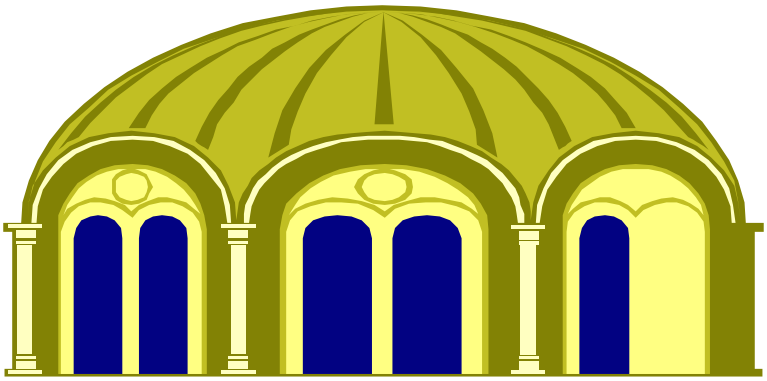
This skills unit contains:
- **features of shape**
- **two dimensional (2D) shapes**
- **three dimensional (3D) shapes**

Features of Shape

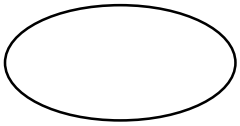
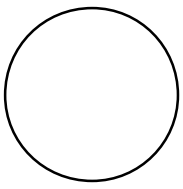
Shapes and symbols are everywhere:
- Some are for communicating eg. A, B, C.....
- Some are for counting eg. 1, 2 , 3 , 4..
- Others are the basic "building blocks" for structures (shapes); they help your understanding of how things fit together.



Shapes are made up of:

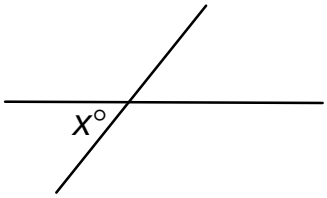
Lines

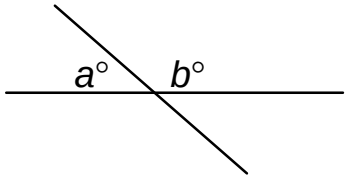
- parts of straight lines (intervals) and combinations of straight lines.
- curves, including closed curves that make regular shapes like the

ellipse  and circle 

Angles

- An angle is the circular measure of the distance between two intervals eg. angles $x°$, $a°$ and $b°$ shown below.

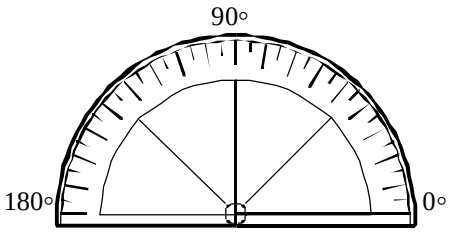




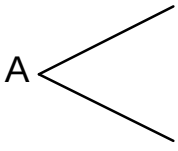
- Many angle properties exist between intersecting lines and parallel lines eg. in the diagram above, angles *a* and *b* form a straight angle of 180°.

- Angles are measured in degrees.

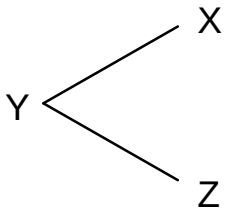
- You use a **protractor to measure angles**.



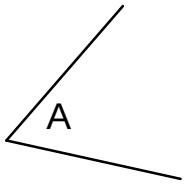
- There are different ways of naming angles:



angle A
or ∠ A



angle XYZ
or ∠ XYZ

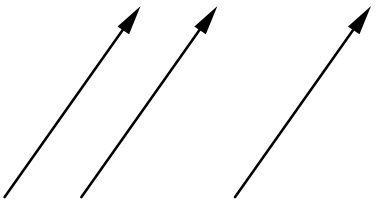


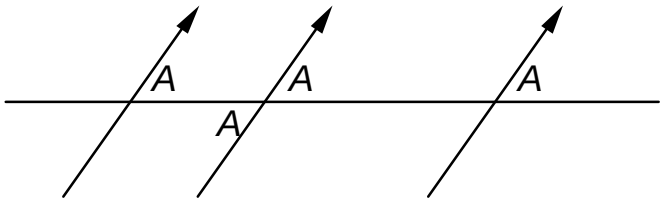
angle A
or ∠ A

Some special lines:

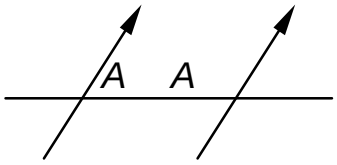
Parallel lines

These are two or more lines with the same direction.

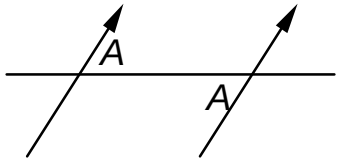




When parallel lines are cut by another line, sets of equal angles are formed. All the ‘A’ angles are equal in the above diagram and the following diagrams:

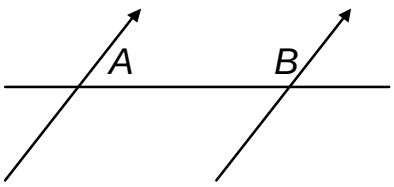


corresponding angles



alternate angles

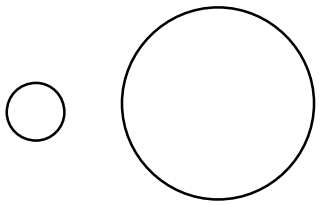
and, $A + B = 180°$; A and B are called co-interior angles



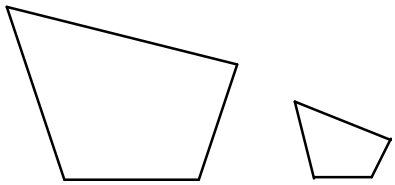
Plane shapes lie completely in one plane (ie. no line 'pokes out' of the plane).

Shape depends on the number and size of the angles, not the lengths of sides. The number of sides of a shape will depend on the number of interior (inside) angles.

Comparing shapes



You need to compare features: the circles have the **same shape** but are **different sizes**.



same shape, different sizes

If figures or objects have the **same shape but are different sizes**, they are said to be **similar**.

Examples of similar shapes:

1 The shape of a **model** of a house is **the same** as the shape of the **actual** or planned house.

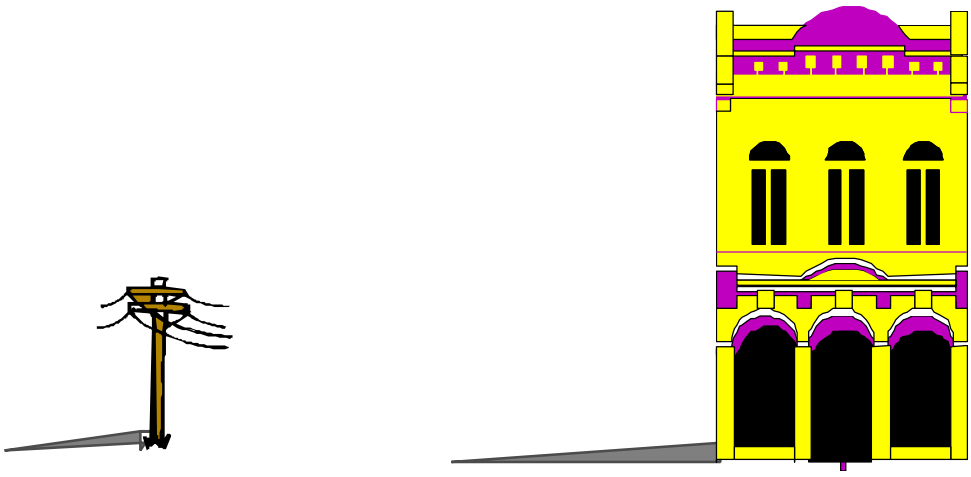
Size of the model may be $\frac{1}{100}$ of the actual size of the house.

Scale factor = 1 : 100 (1 cm on the model represents 100 cm or 1 metre on the house)

2 Local Topographic Maps are similar to the actual area of land and often use a scale of 1 : 100 000

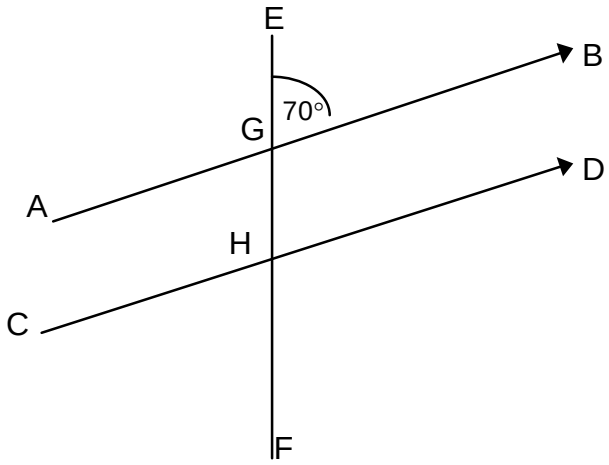
ie. 1 cm $\overset{\text{represents}}{\Rightarrow}$ 100 000 cm or 1000m (1km)

3 A building and a post and their shadows, form two similar shapes at the same time of day.



By measuring the lengths of the two shadows and the height of the post, you can calculate the height of the building (using ratios).
See the **Calculation A** skills unit for more work on ratio.

Activity 1



AB and CD are parallel lines

1 Use a protractor to measure these angles:

∠ EGB = 70°
∠ EHD = ___
∠ AGH = ___
∠ CHF = ___

2 From the four angles listed in 1, find:

a) a pair of alternate angles
b) a pair of corresponding angles

3 Use a protractor to measure these angles:

∠ BGF = ___
∠ DHF = ___

4 Complete:

∠ EGB + ∠ BGF = ___
∠ DHF + ∠ DHE = ___
(These angles are called angles on a straight line)

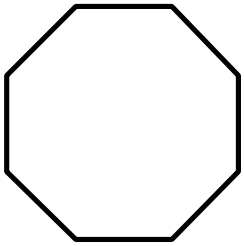
Activity 1 Answers

1 70°, 70°, 70°
2 a) AGH, EHD
b) EGB, EHD or AGH, CHF
3 110°, 110°
4 180°, 180°

Two Dimensional (2D) Shapes

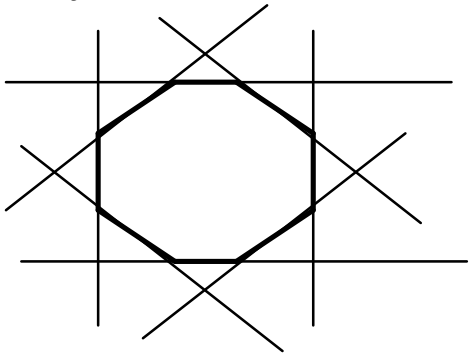
Two dimensional (2D) shapes are shapes that lie in one plane. They are flat with no thickness or depth. Things around you are rarely two dimensional eg. windows, because they do have thickness, however small.

Closed figures with straight line boundaries are called polygons. Each polygon has the same number of sides as it has angles.
eg. An octagon (**oct,** meaning **eight** + **gan** meaning **angle**) has eight angles.



It is common to call an “octagon” an eight sided figure.

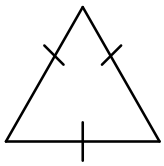
Eight straight lines form the boundaries of the shape. The shape is the space enclosed by the boundaries or line segments.



Common 2D spaces or figures

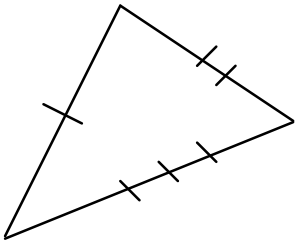
- **Triangles - 3 angles and 3 sides**

1 Equilateral
- all sides have the same length
- all angles are equal (60°)

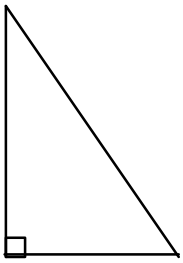


2 Isosceles
- two sides are equal
- two angles (x) are equal

3 Scalene
- three sides are different lengths
- the angles are not equal



4 Right-angled
- one inside angle is a right angle (90°)
- the other two angles total 90°



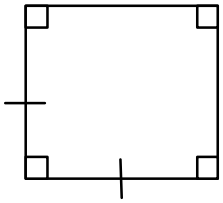
Properties of all triangles:

- the sum of the interior (inside) angles is 180°
- the sum of any two sides is greater than the remaining side.

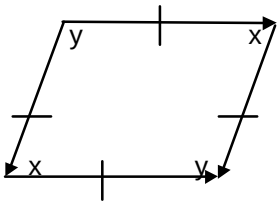
- **Quadrilaterals - 4 angles and 4 sides**

Regular quadrilaterals include:

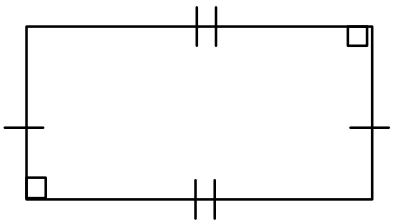
1 Square
- sides are of equal length
- all angles are right angles
- opposite sides are parallel



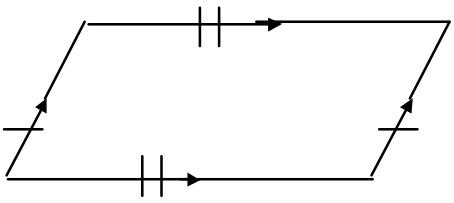
2 Rhombus
- sides are of equal length
- interior angles are not right angles
- opposite angles are equal
- opposite sides are parallel



3 Rectangle
- opposite sides are parallel and of equal length
- all inside angles are right angles

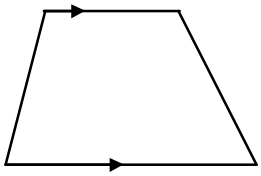


4 Parallelogram
- opposite sides are parallel and of equal length
- opposite angles are equal



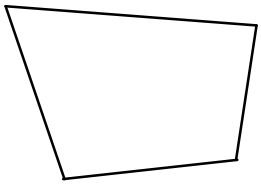
Note: Squares, rhombuses (or rhombi) and rectangles are all parallelograms.

5 Trapezium
- one pair of sides is parallel



Irregular quadrilaterals:

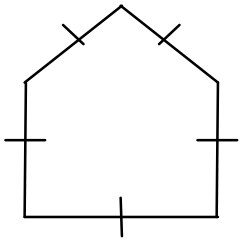
- four-sided shapes
- no sides equal or parallel



Properties of all quadrilaterals:
- they are four-sided, closed figures
- the sum of the interior (inside) angles is 360°

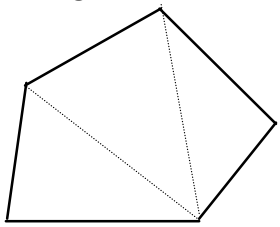
- **Pentagons - 5 angles and 5 sides**

- if regular, the sides are the same length and the angles are the same size



To find the total of the interior angles in a polygon, we usually find the number of triangles which make up the polygon.

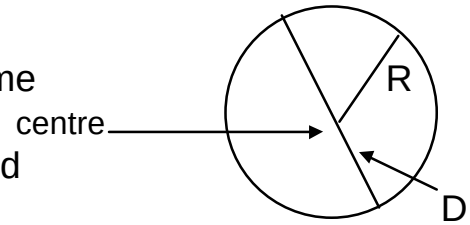
eg. for a pentagon:



three interior triangles, each with 180°
Total of interior angles in a pentagon = 3 x 180° = 540°

- **Circles**

- every point on the circle is the same distance from the centre
- the line through the centre is called the diameter(D)
- the line from the centre to the circle is called the radius (R); radius = $\frac{1}{2}$ diameter
- the circumference is the boundary of the circle



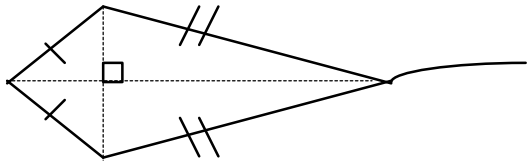
Activity 2

1 Complete this description of a square, beginning:
"a square is a parallelogram with _____ sides equal and _____ _____ a right angle.

2 Write down a similar description of a rectangle.

3 A circle has a diameter of 3.5 m, what is its radius?

4 Use the information shown in the diagram of a kite to give at least two properties of the kite.



5 Complete the table:

No. of sides of polygon	Name of figure	No. of interior angles
6	_____	_____
7	_____	_____
8	_____	_____
9	_____	_____
10	_____	_____

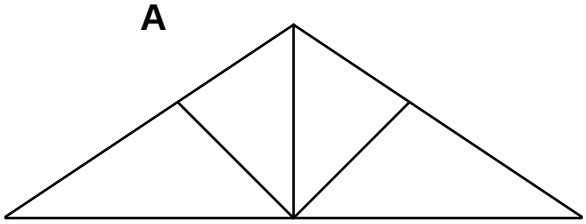
Activity 2 Answers

1 all (or 4), each angle or all angles
2 eg. a rectangle is a parallelogram with opposite sides equal and each angle a right angle
3 1.75 m
4 2 pairs of equal sides, 2 equal angles, the diagonals intersect at right angles
5 hexagon (6), heptagon (7), octagon (8), nonagon (9), decagon (10)

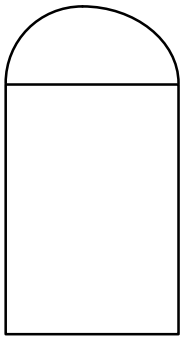
Composite shapes are made up of combinations of simple shapes.

Examples of composite shapes:

A



B



Activity 3

1 a) In the example A above, how many triangles are there?

b) How many quadrilaterals are there altogether in the shapes A and B?

2 In shape B above, what two shapes make up the composite shape?

3 What is the sum of the interior angles of a regular octagon? You might find it helpful to draw a diagram.

(Work out how many triangles make up the inside of the octagon and then use the fact that the sum of the interior angles of a triangle is 180°.)

Activity 3 Answers

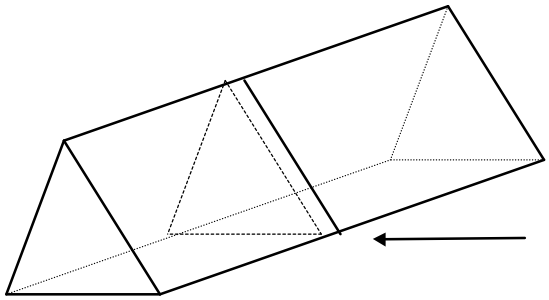
1 a) 7
b) 4
2 rectangle, semicircle
3 1080°

Three Dimensional (3D) Shapes

There are three groups of three dimensional (3D) shapes:

- prisms
- pyramids and
- special shapes like the sphere

Prisms are 3D shapes which have constant cross sections.



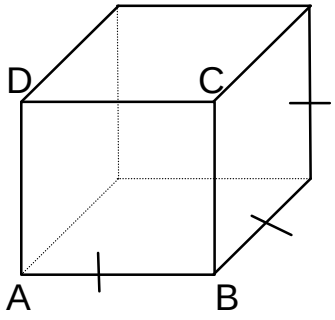
Slices or cross sections taken at any point are the same size and shape.

A prism takes its name from the shape of its base or unchanging cross section.

The diagram above is a **triangular prism** because the shape of its cross-section is a **triangle**.

Common types of prisms:

- **A cube is a square prism**, with height equal to the sides of the square base.



A solid figure has characteristics of:

faces - the surfaces that make up the shape
eg. face ABCD

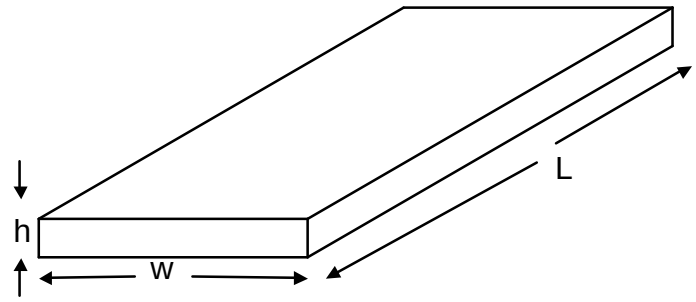
edges - where two faces meet eg. AB and BC are edges

vertices - where 3 or more edges meet
eg. vertex A

The cube has:

- 6 faces
- 12 edges
  - 4 at the ‘top’
  - 4 at the ‘bottom’
  - 4 around the ‘sides’
- 8 vertices
  - 4 at the 'top'
  - 4 at the 'bottom'

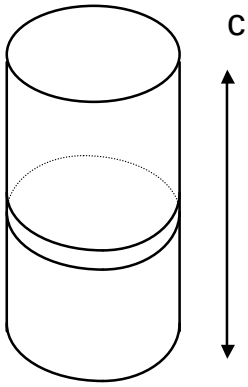
- **Rectangular prism**



The cross section remains a rectangle of size w by h throughout its length or of size w by L throughout its height.

Note: Any shape can form the base of a prism.

- **Circular prism (Cylinder):**



a slice taken at any place over the height, is the same size and shape as the base.

Activity 4

1 Draw prisms on triangular (3 sides)
hexagonal (6 sides)
pentagonal (5 sides)
octagonal (8 sides) bases.

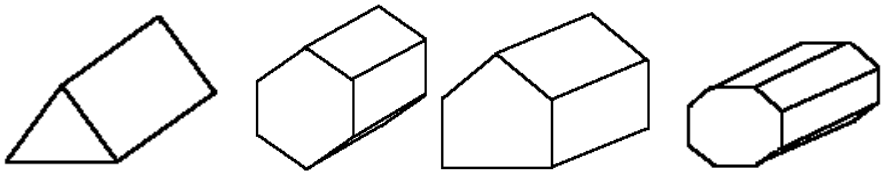
2 Complete the table to give the number of faces, vertices and edges for the prisms you have drawn in 1:

Shape of prism base	**No. of faces**	**No. of vertices**	**No. of edges**
triangle	_____	_____	_____
hexagon	_____	_____	_____
pentagon	_____	_____	_____
octagon	_____	_____	_____

3 Can you see the rule that connects the numbers of faces, vertices and edges?

4 Why is a cylinder different to the prisms above?

Activity 4 Answers

1 

2

triangle	5	6	9
hexagon	8	12	18
pentagon	7	10	15
octagon	10	16	24

3 no. of edges = no. of faces + no. of vertices - 2

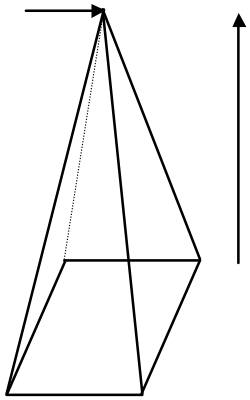
4 no straight edges

Pyramids

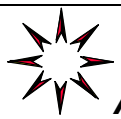
A pyramid, like a prism, takes its name (square, triangular, hexagonal) from the shape of the base (and cross section).

In a pyramid, straight lines from each vertex on the base are joined to a single point or top vertex.

top vertex



The **cross-section shape** remains **the same** as the base but its **size is reduced** as you move from the base to the top vertex.



Activity 5

1 Count the faces, vertices and edges of the square pyramid drawn on the previous page.

2 Draw triangular / hexagonal / pentagonal pyramids.

ie. pyramids with triangular, hexagonal and pentagonal bases.

3 Complete the table of faces, vertices and edges with numbers obtained from the diagrams in questions 1 and 2.

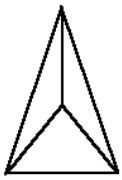
Shape of pyramid base	**No. of faces**	**No. of vertices**	**No. of edges**
square	_____	_____	_____
triangle	_____	_____	_____
hexagon	_____	_____	_____
pentagon	_____	_____	_____

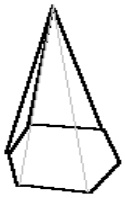
4 Is the rule that connects these numbers the same as the rule for prisms?

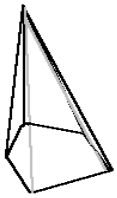
Activity 5 Answers

1 faces : 5
vertices : 5
edges : 8

2





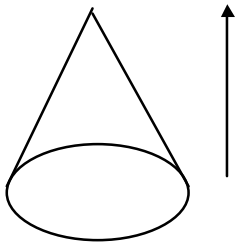


3

square	5	5	8
triangle	4	4	6
hexagon	7	7	12
pentagon	6	6	10

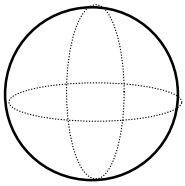
4 yes

A **cone** is a pyramid with a circular base.



the cross section remains circular but becomes smaller as it moves towards the point.

Spheres



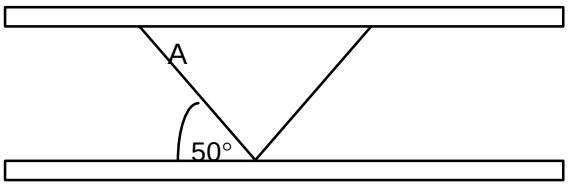
A Sphere is a solid figure with circular cross-sections of the same size when taken through the centre of the sphere.

Note:

- Other characteristics of shape, like surface area and volume, are investigated in the **Measurement B** skills unit.
- The word 'solid' generally refers to a 3 dimensional shape, whether it is 'hollow' or not.
  To avoid confusion, it is common to use terms like:
  - a hollow sphere or
  - a solid sphere.

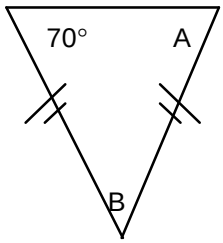
Test yourself on this skills unit

1 A girder is made up of 2 paralel strips of metal, reinforced by cross pieces as shown.
Work out the size of the angle marked A and give a reason for your answer.



2 a) Describe this shape.

b) Use your knowledge of triangles to find the size of angles A and B.



3 Write True or False:

a) all rectangles are parallelograms.

b) a trapezium is a quadrilateral.

c) a rhombus is another name for a square.

d) a circle is a special quadrilateral.

4 a) Draw an irregular hexagon.

b) Divide it into triangles.

c) Find the angle sum of a hexagon.

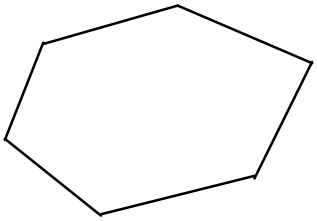
Test yourself on this skills unit Answers

1 A = 50 °
alternate angles, parallel lines

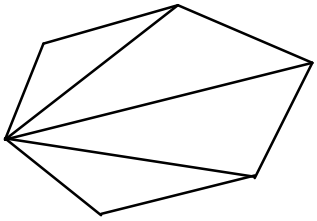
2 a) isosceles triangle
b) A = 70°, B = 40°

3 a) True
b) True
c) False
d) False

4 a)



b)



c) 720°